MATHEMATICS

GRADE 12

PAPER 1		
QUESTION	**TOPICS/CONCEPTS**	**MARKS**
1	**Algebra – Grade 10 & 11 textbooks** • Factorise – all six skills. • Quadratic formula – correct to two decimal places. • Surd and exponential equations • Quadratic Inequality – difference between ***or*** and ***and*** • Simultaneous equations - revise algebraic expressions(products) • Nature of roots	**27**

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 3x - 10 = 0$ (3)

1.1.2 $3x^2 + 6x + 1 = 0$ (correct to TWO decimal places) (3)

1.1.3 $2^{x+4} + 2^x = 8\,704$ (3)

1.1.4 $(x-8)(x+2) \leq 0$ (3)

1.1.5 $x + 3\sqrt{x+2} = 2$ (4)

1.2 A rectangle having sides of $(y - 3)$ metres and $(x + 2)$ metres has a perimeter of 24 metres and an area of 32 square metres. Calculate the values of x and y. (6)

1.3 Show that $\left(1 + x^m + x^{-n}\right)^2 - \left(1 - x^m - x^{-n}\right)^2$ is divisible by 2 for all real values of m and n. (3)

[25]

2	**Number Patterns – Grade 10 textbook** • Linear pattern • Quadratic number patterns	**12**
3	**Number Patterns – Grade 11 textbook** • Arithmetic sequence/series • Geometric sequences/series	**12**
4	**Sequence and Series – Grade 12 textbook** **PROOF OF SUM OF ARITHMETIC AND GEOMETRIC** • Sigma notation • Infinite series • Convergence	**16**

QUESTION 2

2.1 Given the arithmetic series: $7 + 12 + 17 + \ldots$

2.1.1 Determine the value of T_{91} (3)

2.1.2 Calculate S_{91} (2)

2.1.3 Calculate the value of n for which $T_n = 517$ (3)

2.2 The following information is given about a quadratic number pattern:

$T_1 = 3$, $T_2 - T_1 = 9$ and $T_3 - T_2 = 21$

2.2.1 Show that $T_5 = 111$ (2)

2.2.2 Show that the general term of the quadratic pattern is $T_n = 6n^2 - 9n + 6$ (3)

2.2.3 Show that the pattern is increasing for all $n \in N$. (3)
[16]

QUESTION 3

3.1 Given the geometric series: $3 + 6 + 12 + \ldots$ to n terms.

3.1.1 Write down the general term of this series. (1)

3.1.2 Calculate the value of k such that: $\sum_{p=1}^{k} \frac{3}{2}(2)^p = 98\,301$ (4)

3.2 A geometric sequence and an arithmetic sequence have the same first term.

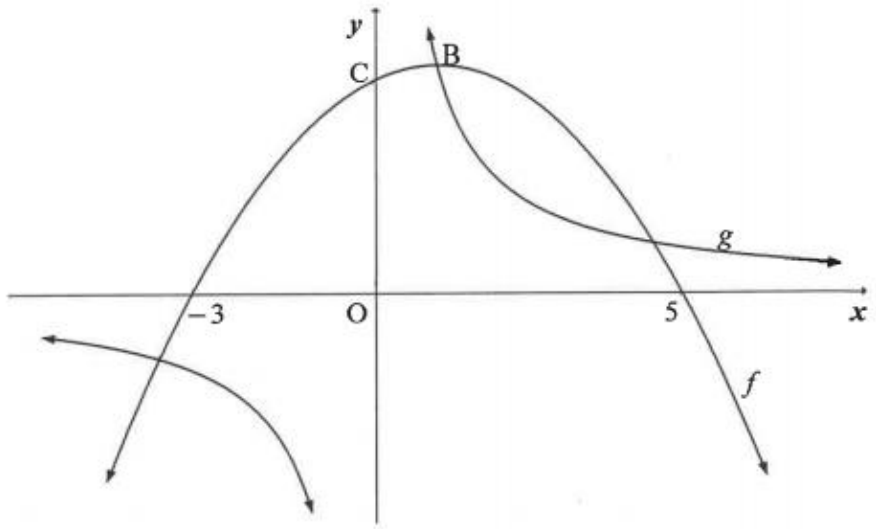
- The common ratio of the geometric sequence is $\frac{1}{3}$
- The common difference of the arithmetic sequence is 3
- The sum of 22 terms of the arithmetic sequence is 734 more than the sum to infinity of the geometric sequence.

Calculate the value of the first term. (5)
[10]

	~~Convergence~~	
5	**Functions - Grade 11** • Linear, Parabola, Hyperbola and Exponential • Intercepts, domain, range, asymptotes, equation of axis of symmetry, T.P, increasing/decreasing interval etc • **ALL** characteristics/ transformations	**13**
6	**Functions – Grade 11** • Linear and hyperbola, exponential • **ALL** characteristics/ transformations $f(x) = g(x)$ • $f(x) \leq g(x)$ $f(x).g(x) > 0$	**14**
7	**Functions – Grade 12** • Inverse Functions • Change from Log to exponential form. • Use a point (on Cartesian plane) to revise Transformation Geometry	**10**

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}(x-1)^2 + 8$ and $g(x) = \frac{d}{x}$ are drawn below. A point of intersection of f and g is B, the turning point of f. The graph f has x-intercepts at $(-3\ ;\ 0)$ and $(5\ ;\ 0)$ and a y-intercept at C.



5.1 Write down the coordinates of the turning point of f. (2)

5.2 Calculate the coordinates of C. (2)

5.3 Calculate the value of d. (1)

5.4 Write down the range of g. (1)

5.5 For which values of x will $f(x).g(x) \leq 0$? (3)

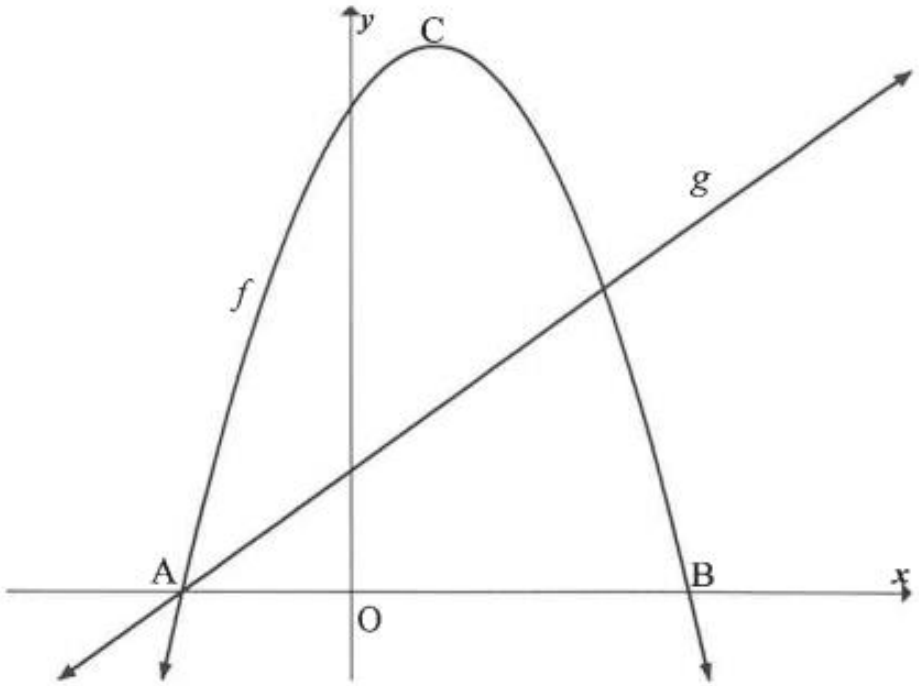
5.6 Calculate the values of k so that $h(x) = -2x + k$ will not intersect the graph of g. (5)

5.7 h is a tangent to g at R, a point in the first quadrant. Calculate t such that $y = f(x) + t$ intersects g at R. (4)

[18]

QUESTION 5

Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$.
A and B are the x-intercepts of f. C is the turning point of f.



5.1 Calculate the coordinates of A and B. (3)

5.2 Determine the coordinates of C, the turning point of f. (2)

5.3 Write down the range of f. (1)

5.4 The graph of $h(x) = f(x + p) + q$ has a maximum value of 15 at $x = 2$.
Determine the values of p and q. (3)

5.5 Determine the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (2)

5.6 For which value(s) of x will $g^{-1}(x).g(x) = 0$? (2)

5.7 If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will NOT intersect. (5)

[18]

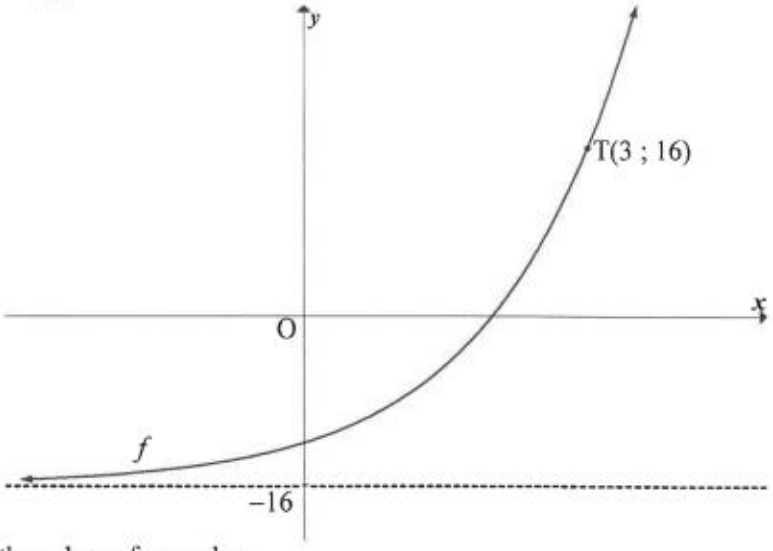
QUESTION 6

6.1 Given: $g(x) = 3^x$

6.1.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

6.1.2 Point P(6 ; 11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinates of the image of P on g. (2)

6.2 Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3 ; 16) is a point on f and the asymptote of f is $y = -16$.

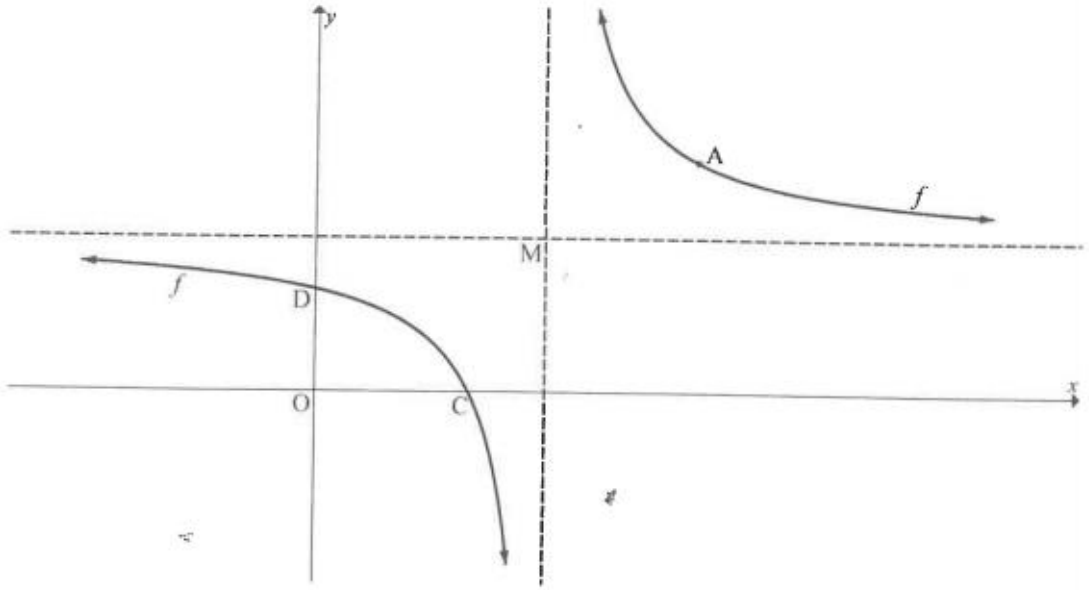


Determine the values of p and q. (4)

[8]

QUESTION 4

The graph of $f(x)=\frac{4}{x-3}+4$ is drawn below. M is the point where the asymptotes of f intersect. C and D are the x- and y-intercepts respectively of f. A is the point on f that is closest to M.



4.1 Write down the coordinates of M. (2)

4.2 Calculate the coordinates of D. (2)

4.3 If $y=x+t$ is the equation of a line of symmetry of f, calculate the value of t. (2)

4.4 Determine the values of x for which $f(x)\leq 0$. (4)

4.5 Calculate the coordinates of A. (3)

4.6 A single transformation is applied to f to obtain a new graph defined as $h(x)=\frac{-4}{x+3}+4$. A$'$ is the image of A under this transformation. Calculate the length of AA$'$. (2)

[15]

8	**Differential Calculus – Grade 12 textbook** • First Principles – Revise algebraic expressions(products) • Differentiation Rules • Equation of the tangent (straight line) • **Revise Exponents and Surds**	**14**
9	**Differential Calculus** • Calculus graph • Definining the turning point and the point of inflection of the cubic graph in relation to the graphs of f' and f'' • Graphs interpretation	**32**
	TOTAL	**150**

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x)=3x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$ if $f(x)=x^2-3+\frac{9}{x^2}$ (3)

8.2.2 $g'(x)$ if $g(x)=\left(\sqrt{x}+3\right)\left(\sqrt{x}-1\right)$ (4)

[12]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x)=-4x^2$ (5)

7.2 Determine:

7.2.1 $f'(x)$ if $f(x)=2x^3-3x$ (2)

7.2.2 $D_x\left(7\sqrt[3]{x^2}+2x^{-5}\right)$ (3)

7.3 For which values of x will the tangent to $f(x)=-2x^3+8x$ have a positive gradient? (3)

[13]

QUESTION 8

Given: $f(x) = -x^3 + 6x^2 - 9x + 4 = (x-1)^2(-x+4)$

8.1 Determine the coordinates of the turning points of f. (4)

8.2 Draw a sketch graph of f. Clearly label all the intercepts with the axes and any turning points. (4)

8.3 Use the graph to determine the value(s) of k for which $-x^3 + 6x^2 - 9x + 4 = k$ will have three real and unequal roots. (2)

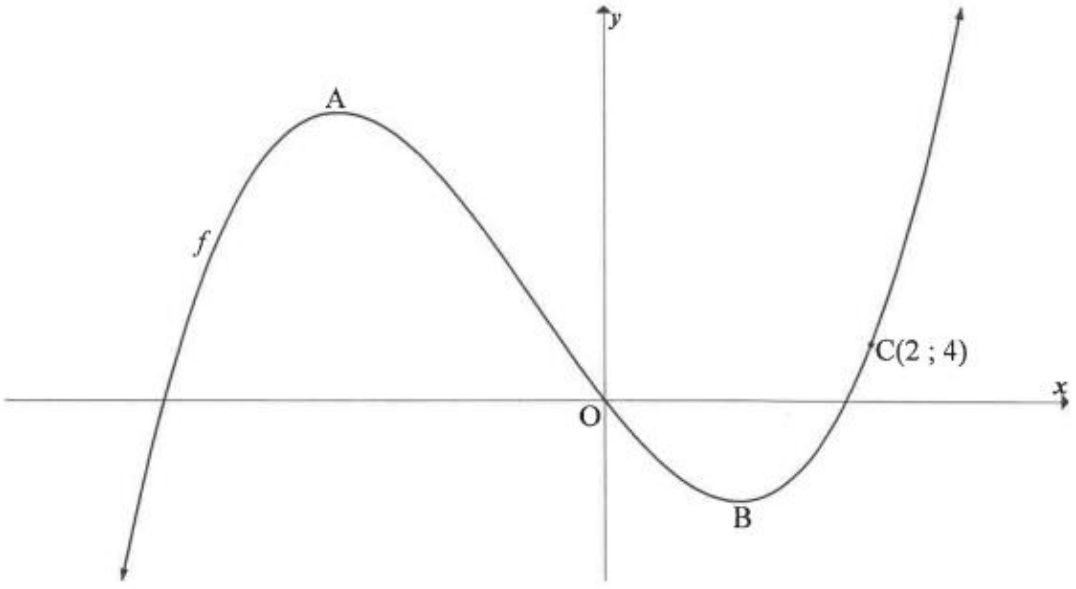
8.4 The line $g(x) = ax + b$ is the tangent to f at the point of inflection of f. Determine the equation of g. (6)

8.5 Calculate the value of θ, the acute angle formed between g and the x-axis in the first quadrant. (2)

[18]

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.
A and B are the turning points of f. C(2 ; 4) is a point on f.



9.1 Determine the coordinates of A and B. (5)

9.2 For which values of x will f be concave up? (3)

9.3 Determine the equation of the tangent to f at C(2 ; 4). (3)

[11]